THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1520C University Mathematics 2014-2015
Suggested Solution to Test 1

1. (a) $\lim_{x\to 5} \dfrac{x^3-125}{x-5} = \lim_{x\to 5} x^2+5x+25 = 75$

   (b) $\lim_{x\to 4} \dfrac{\sqrt{x}-2}{x-4} = \lim_{x\to 4} \dfrac{1}{\sqrt{x}+2} = \dfrac{1}{4}$

2. (a) $\lim_{x\to +\infty} \dfrac{e^x+e^{-x}}{e^x-e^{-x}} = \lim_{x\to +\infty} \dfrac{1+e^{-2x}}{1-e^{-2x}} = 1$

   (b) $\lim_{x\to -\infty} \dfrac{x}{\sqrt{x^2-4x}} = \lim_{x\to -\infty} \dfrac{1}{\frac{1}{x}\sqrt{x^2-4x}} = \lim_{x\to -\infty} \dfrac{1}{-\sqrt{1-\frac{4}{x}}} = -1$

3. We can rewrite $f(x)$ as

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \geq 0 \\ -x^2 & \text{if} \quad x < 0 \end{cases}$$

We have $\lim_{h\to 0^+} f(h) = h^2 = 0$, $\lim_{h\to 0^-} f(h) = -h^2 = 0$ and $f(0) = 0$. Therefore, f is continuous at $x = 0$.

(b) We have

$$\lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{h^2-0}{h} = \lim_{h\to 0^+} h = 0$$

and

$$\lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{-h^2-0}{h} = \lim_{h\to 0^-} -h = 0$$

Therefore, f is differentiable at $x = 0$ and $f'(0) = 0$.

4. We have

$$\begin{aligned} y &= \frac{e^{5x}\sqrt[4]{2x-5}}{(3x-5)^3} \\ \ln y &= 5x + \frac{1}{4}\ln(2x-5) - 3\ln(3x-5) \\ \frac{1}{y}\cdot\frac{dy}{dx} &= 5 + \frac{1}{2(2x-5)} - \frac{9}{3x-5} \\ \frac{dy}{dx} &= \left(5 + \frac{1}{2(2x-5)} - \frac{9}{3x-5}\right)\frac{e^{5x}\sqrt[4]{2x-5}}{(3x-5)^3} \end{aligned}$$

5. (a) The initial population is $N(0) = 5000$.

   (b) We have

$$\begin{aligned} N(t) &= 5000(2+t)e^{-0.01t} \\ N'(t) &= 5000e^{-0.01t}(1-0.01(2+t)) \\ &= 50e^{-0.01t}(98-t) \end{aligned}$$

We then know that $N'(t) > 0$ when $t < 98$ and $N'(t) < 0$ when $t > 98$.

Therefore, the population attains maximum when $t = 98$ and the maximum population is $N(98) = 500000e^{-0.98}$.

(c) $\lim_{t \to +\infty} N(t) = 0$.

6. skipped.